Calculus Review

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Review Notes Pre-Calculus

$\Rightarrow$ Rule or note

Definitions

- Polynomial

Polynomial	Degree	Example
Constant or Zero Polynomial	0	6
Linear Polynomial	1	$3x + 1$
Quadratic Polynomial	2	$4x^2 + 1x + 1$
Cubic Polynomial	3	$6x^3 + 4x^3 + 3x + 1$
Quartic Polynomial	4	$6x^4 + 3x^3 + 3x^2 + 2x + 1$

Simplify: Property of square roots

$\Rightarrow \sqrt{AB} = \sqrt{A}\sqrt{B}$

- Simplify:

$$\sqrt{4x^{10}}$$

- Solution:

$$= \sqrt{4}\sqrt{x^{10}}$$
$$= 2\sqrt{x^{10}}$$
$$= 2x^5$$

Solve for u

$$-\frac{9}{5} = \frac{1}{2}u - \frac{4}{3}$$

First, eliminate the fractions. LCD of 1/2, 4/3, 9/5 is 30

⇒ *The least common denominator (LCD) of two fractions is the least common multiple (LCM) of their denominators.*

$$30(-\frac{9}{5}) = 30(\frac{1}{2}u - \frac{4}{3})$$

Use the *distributive property*:

$$30(-\frac{9}{5}) = 30(\frac{1}{2}u) + 30(-\frac{4}{3})$$

$$(-\frac{30 \times 9}{5}) = (\frac{30 \times 1}{2})u + (-\frac{30 \times 4}{3})$$

$$-54 = 15u - 40$$
$$-14 = 15u$$
$$-\frac{14}{15} = u$$

Radicals

Solve for y:

$$\sqrt{y-13} - 1 = 5$$

First isolate the radical:

$$(\sqrt{y-13})^2 - 1^2 = 5^2$$
$$y - 13 - 1 = 25$$
$$y - 13 = 26$$
$$y = 39$$

Radicals: Multiply

$$\sqrt{2}(\sqrt{3} - 11)$$

distribute:

$$\sqrt{2} \times \sqrt{3} - \sqrt{2} \times 11$$

$\sqrt{AB} = \sqrt{A}\sqrt{B}$

$$\sqrt{2 \times 3} - \sqrt{2} \times 11$$

$$\sqrt{6} - \sqrt{2} \times 11$$

$\sqrt{6}$ and $\sqrt{2}$ are not like radicals, so:

$$\sqrt{6} - 11\sqrt{2}$$

Radicals: No solution

$$12 + \sqrt{v+5} = 4$$

$$\sqrt{v+5} = -8$$

$\Rightarrow$ *The square root of something can not be negative, so this is no solution (ignore $\sqrt{-1}$)*

Unit Circle: Find the exact value of

$$\cos \frac{7\pi}{4}$$

$$(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}) = 315°$$

Domain and range

A relation is a set of *ordered pairs.*

$\Rightarrow$ The *domain* of a relation is the set of all first elements in the ordered pairs.

$\Rightarrow$ The *range* is the set of all second elements in the ordered paris.

Give the domain and range for T (assume using *set notation*):

$$T = \{(0, -9), (7, -1), (-9, 6), (7, 9)\}$$

$$domain = \{0, 7, -9\}$$

$$range = \{-9, -1, 6, 9\}$$

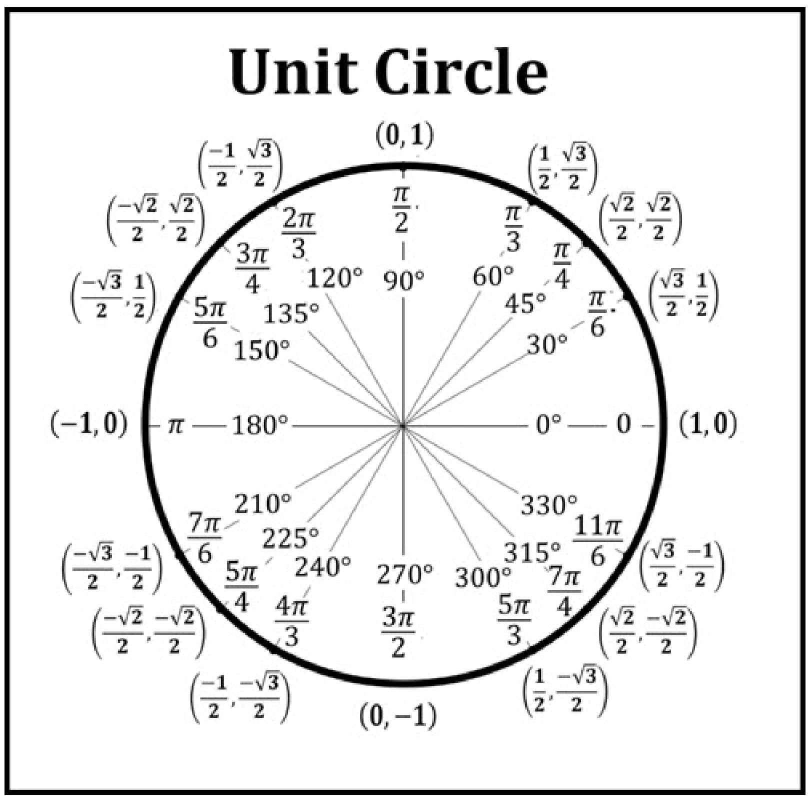

Figure 1: Unit Circle

Solving a rational equation that simplifies to a linear

Solve for u:

$$8 = -\frac{6}{u+8}$$

$\Rightarrow$ **Note**: u can not be -8 because that would make 0 in the denominator.

Get rid of the fractions:

$$\frac{(u+8)}{1} \times 8 = -\frac{6}{u+8} \times (\frac{u+8}{1})$$

$$8(u+8) = -6$$

$$8u + 64 = -6$$

$$8u = -70$$

$$u = \frac{-70}{8}$$

$$u = \frac{-35}{4}$$

Simplifying a ratio of polynomials using GCF

Simplify:

$$\frac{6x^2 - 15x}{3x^2 - 24x}$$

Factor the numerator. GCF of $6x^2$ and $15x$ is $3x$

$$\frac{3x(2x-5)}{3x^2 - 24x}$$

Factor the denominator. GCF of $3x^2$ and $24x$ is $3x$

$$\frac{3x(2x-5)}{3x(x-8)}$$

$$\frac{2x-5}{x-8}$$

Translating a graph

Shift to the right *c* units: $ y = f(x-c) $

Shift to the left *c* units: $ y = f(x+c) $

Shift to the up *c* units: $ y = f(x)+c $

Shift to the down *c* units: $ y = f(x)-c $

Rationalize the denominator

$$\sqrt{\frac{3}{10}}$$

$\Rightarrow$ Quotient property for square roots $\sqrt{\frac{A}{B}} = \frac{\sqrt{A}}{\sqrt{B}}$:

$$\frac{\sqrt{3}}{\sqrt{10}}$$

$\Rightarrow$ Times 1 or $\frac{\sqrt{10}}{\sqrt{10}}$

$$\frac{\sqrt{3}}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$$

$$\frac{\sqrt{3 \times 10}}{\sqrt{10 \times 10}}$$

$$\frac{\sqrt{30}}{10}$$

Completing the square

Fill in the blank to make the expression a perfect square:

$$v^2 - 12v + \square$$

Find $\frac{1}{2}$ of the coefficient of v: $\frac{1}{2} \times (-12) = -6$

Square the result of the last step: $(-6)^2 = 36$

$$v^2 - 12v + 36$$

Find slope given two points

Find slope of points: $(-7, 8)$ and $(5, -7)$ $((x, y))$

$\Rightarrow$ $slope = \frac{rise}{run}$ or $slope = \frac{y_2 - y_1}{x_2 - x_1}$

$$\frac{-7-8}{5+7}$$
$$\frac{-15}{12}$$
$$-\frac{5}{4}$$

Factor: difference of squares

$\Rightarrow A^2 - B^2 = (A+B)(A-B)$

Factor:

$$49 - 64x^2$$

$$7^2 - (8x)^2$$
$$(7+8x)(7-8x)$$

Rewrite the expression without an exponent

$$\frac{1}{2x^{-4}}$$

$\Rightarrow a^{-n} = \frac{1}{a^n}$

$\Rightarrow \frac{1}{a^{-n}} = a^n$

$$\frac{x^4}{2}$$

Factor out a constant before a quadratic

$$4x^2 - 12x - 72$$

Factor out a 4:

$$4(x^2 - 3x - 18)$$

⇒ Quadratic "plus times thing"... What two numbers multiply together to give -18 (the third number in the equation above), and also add together to give 3 (the second number in the equation above)?

$p + q = \square$

$p * q = \square$

It's -6 and 3. $-6 + 3 = 3$ and $-6 \times 3 = -18$.

You then use those to to fill out the *difference of squares* bits:

$$4(x + 3)(x - 6)$$

Graph the function

$$f(x) = -4x^2 + 5$$

Feed in values starting from -2 and go to 2 to get a few points:

x	$-4x^2 + 5$	$(x, f(x))$
-2	$-4(-2)^2 + 5$	$(-2, -11)$
-1	$-4(-1)^2 + 5$	$(-1, 1)$
$ 0$	$-4(0)^2 + 5$	$(0, 5)$
$ 1$	$-4(1)^2 + 5$	$(1, 1)$
$ 2$	$-4(2)^2 + 5$	$(2, -11)$

The answer to this equation is a *parabola* with it's *vertex* at $(0, 5)$

Roots of a quadratic equation with leading coefficient

Solve for w:

$$3w^2 - 2 = -w$$

Make it equal to zero:

$$3w^2 + w - 2 = 0$$

⇒ Use the *quadratic "plus times thing"* (see above), but we have to get the first term's 3 involved now: $-2 * 3 = -6$ and $-2 + 3 = 1 \ldots\ -6, 1$

$p * q = -6$

$p + q = -6$

$$(3w - 2)(w + 1) = 0$$

Would be true only if one of those above equaled 0 (zero times something equals zero). So, $3w - 2 = 0$ or $w + 1 = 0$.

$$w = \frac{2}{3}, -1$$

Multiply polynomial (Quadratic times Linear)

$$(6v - 2y + 5)(v - 7)$$

Distribute:

$$6v(v - 7) - 2y(v - 7) + 5(v - 7)$$

$$6v^2 - 42v - 2vy + 14y + 5v - 35$$

Combine like terms:

$$6v^2 - 37v - 2vy + 14y - 35$$

Factor multi-variable addition

$$30u^3w^5x^6 + 24u^7w^8$$

⇒ GCF = $6u^3w^5$ - the smallest variable is the greatest common factor.

$$6u^3w^5(5x^6) + 6u^3w^5(4u^4w^3)$$

$$6u^3w^5(5x^6 + 4u^4w^3)$$

Simplify quadratic fraction

$$\frac{3w^2 + 9w + 6}{w^2 + 5w + 6}$$

Factor the numerator - pull out a 3:

$ = 3(x^{2}+3w+2) $

Do the (*plus times thing*):

$ = 3(w+1)(w+2) $

Factor the denominator (*plus times thing*):

$ = (w+3)(w+2) $

$$\frac{3(w+1)(w+2)}{(w+3)(w+2)}$$

Cross out like terms:

$$\frac{3(w+1)}{(w+3)}$$

Multiply quadratic and linear fraction

$$\frac{x^2 - 1}{x + 2} \times \frac{3x + 6}{x^2 - 4x + 3}$$

Simplify terms:

$\Rightarrow x^2 - 1 = (x+1)(x-1)$

$$\frac{(x+1)(x-1)}{x+2} \times \frac{3(x+2)}{(x-1)(x-3)}$$

Remove like terms (cross):

$$\frac{(x+1)}{1} \times \frac{3}{(x-3)}$$

Multiply:

$$\frac{(x+1)}{1} \times \frac{3}{(x-3)} = \frac{3(x+1)}{x-3}$$

Simplify:

$$\frac{3x+3}{x-3}$$

Divide quadratic and linear fraction

$$\frac{4x+24}{5} \div \frac{8x+48}{3}$$

Multiply by the reciprocal:

$$\frac{4x+24}{5} \times \frac{3}{8x+48}$$

Simplify:

$$\frac{4(x+6)}{5} \times \frac{3}{8(x+6)}$$

Cross out like terms, multiply, simplify:

$$\frac{4}{5} \times \frac{3}{8}$$
$$\frac{12}{40}$$
$$\frac{3}{10}$$

Least Common Multiple (LCM)

Find the LCM of $9c^3$ and $6c$

$$18c^3$$

⇒ The LCM is the first number both of the integer parts can both divide into, and the variable part is the largest one in the set.

Least Common Denominator (LCD)

Find the LCD of: $\frac{1}{x-3}$ and $\frac{8}{x+3}$

⇒ Find the LCM of the denominator. Since there isn't one in this example, you take the product.

$$(x-3)(x+3)$$

Equivalent ration expressions

Fill in the blank to make equivalent ration expressions:

$$\frac{2w}{w-7} = \frac{\square}{(w-7)(w-3)}$$

Note that $(w-3)$ is missing from the starting fractions denominator. Multiply by 1 to fix that:

$$\frac{2w}{w-7} \times \frac{(w-3)}{(w-3)} = \frac{2w^2-6}{(w-7)(w-3)}$$

$$\square = 2w^2 - 6$$

Subtract fraction with variables

$$\frac{5}{3} - \frac{7}{4c}$$

⇒ Denominators must be the same; LCD $= 12c$

$\frac{5}{3} \times \frac{4c}{4c} = \frac{20c}{12c}$

$\frac{7}{4c} \times \frac{3}{3} = \frac{21}{12c}$

$$\frac{20c}{12c} - \frac{21}{12c} = \frac{20c-21}{12c}$$

Subtract linear fraction

$$\frac{8c+3}{3c} - \frac{7c+2}{9c}$$

LCD $= 9c$

$\frac{8c+3}{3c} \times \frac{3}{3} = \frac{24c+9}{9c}$

$$\frac{24c+9}{9c} - \frac{7c+2}{9c}$$

$$\frac{17c+7}{9c}$$

Sine Cosine Tangent (SOH CAH TOA)

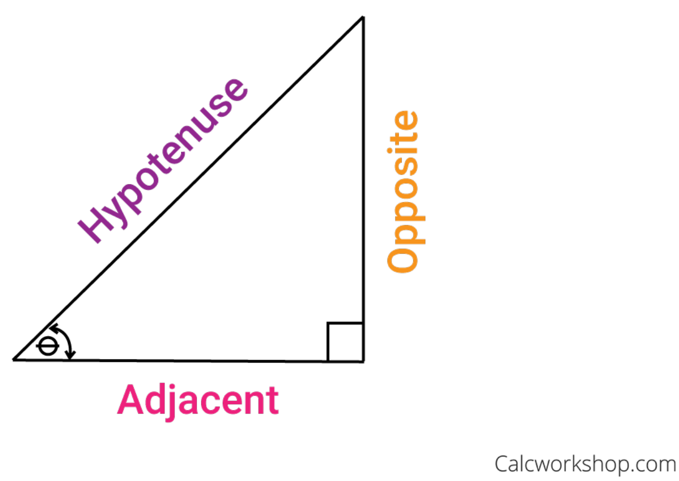

Figure 2: Opposite Adjacent Hypotenuse

$\Rightarrow a^2 + b^2 = c^2$

Add linear fraction

$$\frac{7}{x+3} + \frac{6}{x-2}$$

LCD $= (x+3)(x-2)$

$\frac{7}{x+3} \times \frac{x-2}{x-2} = \frac{7(x-2)}{(x+3)(x-2)}$

$\frac{6}{x-2} \times \frac{x+3}{x+3} = \frac{6(x+3)}{(x+3)(x-2)}$

$$\frac{7(x-2)}{(x+3)(x-2)} + \frac{6(x+3)}{(x+3)(x-2)}$$

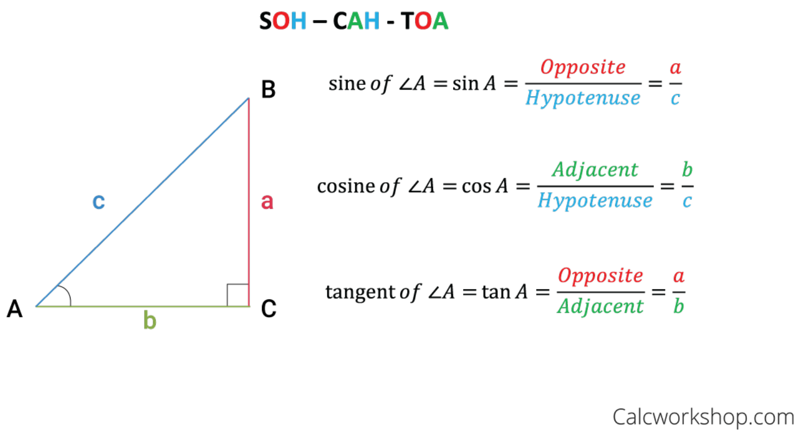

Figure 3: SOH CAH TOA

$$\frac{7x-14}{(x+3)(x-2)} + \frac{6x+18}{(x+3)(x-2)}$$

$$\frac{13x+4}{(x+3)(x-2)}$$

Divide fractions

$$\frac{\frac{10}{9}}{\frac{5}{2}-4}$$

Simplify the denominator. Use LCD

$\frac{5}{2} - (\frac{4}{1} \times \frac{2}{2}) = \frac{5}{2} - \frac{8}{2} = -\frac{3}{2}$

$$\frac{\frac{10}{9}}{-\frac{3}{2}}$$

Multiply by the reciprocal instead:

$$\frac{10}{9} \times -\frac{2}{3}$$

$$-\frac{20}{27}$$

Divide linear fractions

$$\frac{\frac{x-3}{x^5}}{\frac{x-3}{x}}$$

Rewrite as division to not blow your mind.

$$\frac{x-3}{x^5} \div \frac{x-3}{x}$$

Reciprocal to make it multiplication:

$$\frac{x-3}{x^5} \times \frac{x}{x-3}$$

Cancel common factors:

$$\frac{1}{x^4} \times \frac{1}{1}$$

$$\frac{1}{x^4}$$

Radical expressions

Write the following as a radical expression:

$$14\frac{2}{5}$$

$\Rightarrow$ Rules of exponents: $x\frac{m}{n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

$\Rightarrow$ $\sqrt[n]{x^m}$ must be a real number always when n is odd. When it is event only when x >= 0.

$$\sqrt[5]{14^2}$$

Simplified radical form Write in simplified radical form:

$$\sqrt[3]{81}$$

Cube root, focus on perfect cubes...

$8 = 2^3$	$16 = 2^4$	$32 = 2^5$
$27 = 3^3$	$81 = 3^4$	$243 = 3^5$

$64 = 4^3$	$256 = 4^4$	$\dots$
$125 = 5^3$	$\dots$	$\dots$

Write the perfect cube root as a factor

$$\sqrt[3]{27 \times 3}$$

Rewrite 27 as a cube root (from the table above)

$$\sqrt[3]{3^3 \times 3}$$

$\Rightarrow$ Using the rule $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$

$$\sqrt[3]{3^3} \times \sqrt[3]{3}$$

$\Rightarrow$ Using the rule $\sqrt[n]{a^n} = |a|$ for positive a

$$3 \times \sqrt[3]{3}$$

$$3\sqrt[3]{3}$$

Notes

$$\int_a^b f(x)dx$$

$$\iiint f(x,y,z)dxdydz$$

$$\vec{v} =< v_1, v_2, v_3 >$$

$$\vec{v} \cdot \vec{w}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$
